



Unione
Matematica
Italiana



SBM
SOCIEDADE BRASILEIRA DE MATEMÁTICA



SIMAI
SOCIETÀ ITALIANA DI MATEMATICA
APPLICATA E INDUSTRIALE



SBMAC
Sociedade Brasileira de Matemática
Aplicada e Computacional



Università
degli Studi di
Messina

BOOK OF ABSTRACTS

SS12 - Topological methods for the qualitative
analysis of differential equations

2nd Joint Meeting Brazil-Italy
in Mathematics

September 10 and 11, 2026

Messina, Italy

Organizers:

Pierluigi Benevieri, University of São Paulo

Alessandro Calamai, Polytechnic University of Marche

Gennaro Infante, University of Calabria

Periodic solutions for second order time and state-dependent delay equations with a non-asymptotic condition

*Pablo Amster, Pierluigi Benevieri*¹

Department of Mathematics, University of São Paulo, USP

Pablo Amster

Department of Mathematics, University of Buenos Aires

We investigate the existence of periodic solutions, of a given period T , to a class of non-autonomous second order differential equations with time and state-dependent delay. We assume that the associated linear part contains a friction term and the nonlinearity satisfies a non-asymptotic condition. We present some applications to superlinear problems and to a sunflower-type equation with variable delay. The approach is topological, based on Mawhin's coincidence degree. This is a joint work with Pablo Amster.

References

- [1] P. Amster, P. Benevieri *Periodic solutions for second order time and state-dependent delay equations with a non-asymptotic condition*, preprint.

¹The authors are partially supported by FAPESP, project n. 2024/21356-2
E-mail: pluigi@ime.usp.br.

Functional Inequalities with Boundary Terms

*João Marcos Bezerra do Ó*¹

Department of Mathematics, Federal University of Paraíba, Brazil

Weighted Hardy–Friedrichs and Hardy–Sobolev type inequalities with boundary terms in unbounded domains are studied. Based on these inequalities, Liouville-type theorems are established, and applications to quasilinear elliptic problems are discussed.

Non-Absolute Integration and Random Dynamics

*Márcia Federson*¹

Department of Mathematics, University of São Paulo

We review the development of generalized differential equations arising from non-absolute integration, beginning with deterministic systems and extending to stochastic and random dynamics. The talk highlights how non-absolute integration provides a natural framework for modeling irregular phenomena beyond the scope of classical differential equations.

¹Supported by CNPq and FAPESP.
E-mail: `address@server.it`.

Continuation theorems for periodic systems and applications to problems with nonlinear time-dependent differential operators

Pierluigi Benevieri

Instituto de Matemática, Estatística e Ciência da Computação
Universidade de São Paulo, Brazil

Guglielmo Feltrin

Dipartimento di Scienze Matematiche, Informatiche e Fisiche
Università degli Studi di Udine, Italy

We present some continuation theorems for the periodic problem

$$\begin{cases} x'_i = g_i(t, x_{i+1}), & i = 1, \dots, n-1, \\ x'_n = h(t, x_1, \dots, x_n), \\ x_i(0) = x_i(T), & i = 1, \dots, n, \end{cases}$$

providing a unified framework that improves and extends earlier contributions by Jean Mawhin and collaborators to second-order differential problems governed by nonlinear time-dependent differential operators of the form

$$\begin{cases} (\phi(t, x'))' = f(t, x, x'), \\ x(0) = x(T), \quad x'(0) = x'(T). \end{cases}$$

Our approach relies on topological degree theory. As an application, we also establish a Hartman-type theorem for periodic solutions.

References

- [1] P. Benevieri, G. Feltrin, *Continuation theorems for periodic systems and applications to problems with nonlinear time-dependent differential operators*, J. Differential Equations 476 (2026), Paper No. 114464, 25 pp.

A topological degree for studying epidemic models with age-structured population

*Luisa Malaguti*¹

Department of Sciences and Methods for Engineering, University of Modena and Reggio Emilia

Stefania Perrotta

Department of Physics, Informatics and Mathematics, University of Modena and Reggio Emilia

This talk focuses on the study of an age-structured epidemic model, in which a population affected by a disease is divided into susceptible, infected, and recovered individuals. The model, originally introduced by Iannelli and his collaborators [2], is naturally formulated in an abstract functional setting and has traditionally been analyzed using fixed-point techniques, frequently the Banach contraction principle. Following Inaba [3], we instead adopt a semigroup approach and extend, in this context, earlier results by Banasiak [1]. The main novelty of our work is the use of the topological degree for condensing maps instead of classical fixed-point arguments. This alternative approach also allows us to incorporate a nonlinear, time-dependent force of infection; consequently, we can prove the existence of a unique, global, nonnegative solution taking values in the L^1 space with respect to the age variable and satisfying the prescribed initial and standard nonlocal conditions. These results are based on [4].

References

- [1] J. Banasiak *Transport processes with coagulation and strong fragmentation*, Discrete Contin. Dyn. Syst. Ser. B, 17, (2012)
- [2] M. Iannelli, *Mathematical Theory of Age-Structured Population Dynamics*, Applied Mathematics Monographs 7, Giardini (Pisa) (1995)
- [3] H. Inaba, *Age-structured population dynamics in demography and epidemiology*, Springer (Singapore), (2017)
- [4] L. Malaguti and S. Perrotta *A topological degree for studying epidemic models with age-structured population*, submitted, (2026)

¹Aknowledgement: PRIN 2022 project *Modeling, Control and Games through Partial Differential Equations*, funded by the European Union - Next Generation EU, Mission 4, Component 1, CUP D53D23005620006.

E-mail: luisa.malaguti@unimore.it.

Global positive increasing solutions for equations with double-phase operator

Zuzana Došlá

Department of Mathematics and Statistics, Masaryk University, Czech Republic

Mauro Marini, Serena Matucci

Department of Mathematics and Computer Science, University of Florence, Italy

The existence of global positive increasing solutions, with different asymptotic behavior, is investigated for difference equations with double-phase operator

$$\Delta(a(n)\Phi_T(\Delta x(n))) + b(n)f(x(n+1)) = 0,$$

where Δ is the forward difference operator $\Delta x(n) = x(n+1) - x(n)$, Φ_T is given by

$$\Phi_T(u) = |u|^{\alpha-1}u + |u|^{\beta-1}u, \quad 0 < \alpha < \beta,$$

the sequences a, b are positive for $n \in \mathbb{N}$, with

$$\sum_{k=1}^{\infty} \left(\frac{1}{a(k)} \right)^{-1/\alpha} = \infty,$$

and f is a real continuous function such that $vf(v) > 0$ for $v \neq 0$.

Unbounded solutions are classified as regular solutions, if they are either weakly increasing or linearly increasing, and unregular ones. Necessary and/or sufficient conditions for the existence of all these types of solutions are given, using a half-linearization technique and a fixed-point approach, where the fixed point operator is defined through an appropriate disconjugated half-linear equation. The global positivity for these solutions, that is their positivity on a fixed a-priori unbounded interval, is one of the main nontrivial problems. The approach uses some peculiarities of the discrete setting and exhibits significant differences from the continuous case. These discrepancies are also discussed, together with some examples.

References

- [1] Z. Došlá, M. Marini, S. Matucci, *Boundary value problems for equations with double-phase operator*, preprint.
- [2] Z. Došlá, M. Marini, S. Matucci, *Discrete functional equations with in- homogeneous operator and multiple delays*, preprint.
- [3] Z. Došlá, M. Marini, S. Matucci, A Dirichlet problem on the half-line for nonlinear equations with indefinite weight, *Ann. Mat. Pura Appl.* (4) 196 (2017), 51–64.
- [4] O. Došlý O., P. Řehák, *Half-linear Differential Equations*, North-Holland, Mathematics Studies 202, Elsevier Sci. B.V., Amsterdam 2005.

A competitive chemostat model with time-dependent delays

Teresa Faria

University of Lisbon, Portugal

Jaqueline Mesquita

State University of Campinas, Brazil

A non-autonomous chemostat model with time-dependent delays modelling n microorganisms in competition is derived and studied. Under very mild general conditions on the coefficients and time-varying delays, we study the extinction of all the species and, in the case of a periodic system, the existence of nontrivial and nonnegative periodic solutions. For the model with a simple microorganism, a criterion for the uniform persistence is established, which also implies the global attractivity of any positive solution. In this way, a criterion for the existence, uniqueness and global attractivity of a positive periodic solution is derived. These results largely generalize and enhance recent achievements in the literature.

Optimal Periodic Strategies for Integro-Differential Equations with Feedback Controls and Nonlocal Spatiotemporal Dispersal

Paola Rubbioni

Department of Mathematics and Computer Science, University of Perugia

The topic of this talk is our recent investigation into optimal strategies for a nonlinear integro-differential system incorporating both spatial and temporal nonlocal terms, with feedback-based spatially nonlocal controls, and subject to periodic conditions in time. Arising in population dynamics, the model describes the evolution of population density under growth, migration, and memory effects. We discuss existence results for controlled trajectories that approximate the infimum of a given cost functional with arbitrary precision and, under additional regularity assumptions, for truly optimal trajectories. The analysis is carried out using topological methods in nonlinear analysis.

References

- [1] P. Rubbioni, *Existence of optimal periodic strategies in a model with nonlocal spatiotemporal dispersal*, J. Math. Anal. Appl. 556 (2026) 130095

Exploring set-valued selection problems: from fixed points to differential inclusions

Domenico Scopelliti

Department of Economics Statistics and Finance, University of Calabria

A fundamental problem of set-valued analysis is to determine under which assumptions a set-valued map admits a selection (either a single-valued or a set-valued), that is, a map whose graph is contained in the graph of a given set-valued map. A first answer is given by the Axiom of Choice that always guarantees the existence of selections: however, they may have no useful properties. In his pioneering works, E. Michael provided a series of results on the existence of continuous selections of a suitable lower semicontinuous set-valued map. On the other hand, in general, it is not possible to guarantee the existence of continuous (single-valued) selections for upper semicontinuous set-valued maps. Nevertheless, approximate selections has been considered by A. Cellina for this class of set-valued maps.

In the last decades, there was an increased interest in generalizing these selection theorems: in this talk, our aim is to go further by weakening some classical requirements in both cases. To support our results, we analyze some mathematical problem arising from fixed points and differential inclusions.

References

- [1] J. Aubin, A. Cellina, *Differential Inclusions: Set-Valued Maps and Viability Theory*, Springer Science, 1984.
- [2] D. Aussel, A. Pezzola, R. Riccardi, D. Scopelliti *An alternative to the convexity of values in set-valued analysis*, To appear.
- [3] G. Infante, D. Scopelliti *Approximable set-valued selections*, Work in progress.

An affine Birkhoff–Kellogg type result in product spaces and its application to differential systems

Alessandro Calamai

Department of Construction, Civil Engineering and Architecture, Polytechnic University of Marche

Gennaro Infante

Department of Mathematics and Computer Science, University of Calabria

*Giuseppe Antonio Veltri*¹

Department of Mathematics and Computer Sciences, Physical Sciences and Earth Sciences, University of Messina

We obtain a new Birkhoff–Kellogg type result on the product space of translates (in four different versions) regarding the existence and localization of solutions. The theorem is then applied to systems of delayed differential equations.

References

- [1] G. D. Birkhoff, O. D. Kellogg, *Invariant points in function space*, Trans. Amer. Math. Soc., 23 (1922), 96–115.
- [2] A. Calamai, G. Infante, *An affine Birkhoff–Kellogg type result in cones with applications to functional differential equations*, Math. Meth. Appl. Sci., 46 (2023), no. 11, 11897–11905.
- [3] A. Calamai, G. Infante, J. Rodríguez-López, *Birkhoff–Kellogg type results in product spaces and their application to differential systems*, J. Fixed Point Theory Appl, 28 (2026), 37.
- [4] D. Guo, V. Lakshmikantham, *Nonlinear problems in abstract cones*, Academic Press, Boston 1988.
- [5] A. Granas, J. Dugundji, *Fixed Point Theory*, Springer, New York, 2003.
- [6] G. Infante, *Eigenvalues of elliptic functional differential systems via a Birkhoff–Kellogg type theorem*, Mathematics, 9 (2021), n. 4.
- [7] M. A. Krasnosel’skiĭ, *Positive solutions of operator equations*, Noordhoff, Groningen, 1964.
- [8] M. A. Krasnosel’skiĭ, L. A. Ladyženskii, *The structure of the spectrum of positive nonhomogeneous operators*, Trudy Moskov. Mat. Obšč, 3 (1954), 321–346.

¹The authors are members of the Gruppo Nazionale per l’Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM). G. Infante and G. A. Veltri are members of the UMI Group TAA. G. Infante is a member of the “The Research Italian network on Approximation (RITA)”.

E-mail: giuseppe.veltri@studenti.unime.it.