BOOK OF ABSTRACTS

SS30 - Variational and topological methods for local and nonlocal problems

2nd Joint Meeting Brazil-Italy in Mathematics

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Organizers:

Umberto Guarnotta, University of Catania
Salvatore Angelo Marano, University of Catania
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SS30

On the Existence of Multiple Solutions for Singular Integro-Differential Equations Driven by the Fractional Laplacian

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In this paper we investigate the existence and multiplicity of solutions for a class of singular elliptic equations driven by the fractional Laplacian operator. Due to the presence of the singular term, we employ a robust approach combining the sub-supersolution method with variational techniques. By applying the Mountain Pass Theorem to a suitably truncated energy functional, we establish the existence of at least two positive solutions. Our results extend classical multiplicity results for singular elliptic problems to the nonlocal framework, thereby contributing to the development of the theory of integro-differential equations.

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A critical Neumann problem with anisotropic p-laplacian

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The aim of this work is to extend some results obtained by Wang **[1]** to the framework of anisotropic p-Laplace type operators. More precisely, we study the critical Neumann problem

$$\begin{cases} -\Delta_p^H u + |u|^{p-2}u = \lambda |u|^{q-2}u + |u|^{p^*-2}u & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ a(\nabla u)\cdot \nu = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Delta_p^H u = \operatorname{div}\left(H(\nabla u)^{p-1}\nabla H(\nabla u)\right)$ denotes the anisotropic p-Laplacian. Here $H : \mathbb{R}^N \to \mathbb{R}$ is a positive function of class $C^0(\mathbb{R}^N) \cap C^2(\mathbb{R}^N \setminus \{0\})$ satisfying appropriate convexity and homogeneity assumptions, $1 < p < N$, $p < q < p^*$ with $p^* = \frac{Np}{N-p}$ being the critical Sobolev exponent, and $\lambda > 0$ is a parameter.

The domain Ω satisfies the geometric condition:

$$\Omega \subset \Sigma \text{ is a } C^1 \text{ bounded domain inside a convex open cone } \Sigma \text{ in } \mathbb{R}^N, \tag{2}$$

with $\partial\Omega \cap \partial\Sigma$ being a C^1-manifold, and ν is the unit outward normal to $\partial\Omega$. The boundary condition $a(\nabla u)\cdot\nu = 0$, where $a(\xi) = H(\xi)^{p-1}\nabla H(\xi)$, represents the natural Neumann condition associated with the anisotropic operator.

1. Main Results

The main result of this work reads as follows:

Theorem 1. *Assume that $\Omega \subset \mathbb{R}^N$ satisfies (2) and let $H : \mathbb{R}^N \to \mathbb{R}$ be a positive function of class $C^0(\mathbb{R}^N) \cap C^2(\mathbb{R}^N \setminus \{0\})$ satisfying hypotheses $(H1) - (H3)$. Let $1 < p < N$ and $p < q < p^*$. Then the following assertions hold:*

- **(i)** *Let $N \geq p^2$. Then there exists at least one solution of (1) for all $\lambda > 0$.*
- **(ii)** *Let $N = p^2 - p + 1$. Then there exists at least one solution of (1) for $p < q < p^*$ for all $\lambda > 0$ sufficiently large. If $(p-1)N/(N-p) < q < p^*$ then a solution of (1) exists for all $\lambda > 0$.*

Moreover, the solution of (1) obtained belongs to $C^{1,\alpha}(\Omega)$ for some $\alpha \in (0,1)$.

A challenging step in the proof of this theorem is to show that the weak limit u of a Palais–Smale sequence of the associated energy functional J is indeed a critical point. To overcome the lack of compactness inherent to the anisotropic and critical setting, we employ a result of Boccardo and Murat **[2]** to establish almost everywhere convergence of the gradients of the Palais–Smale sequence.

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Making a *Brunelleschi* out of it: concavity in PDEs

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In this talk we will discuss *concavity* properties of solutions to some Partial Differential Equations of the type $-\Delta u = f(u)$, ranging from weaker notions to error estimates and bad optimizers.

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Existence and behavior of positive solutions for elliptic problems with discontinuous nonlinearities via nonsmooth penalization

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In this talk, we present existence and asymptotic behavior results for positive solutions of a class of nonlinear elliptic problems involving discontinuous nonlinearities of Heaviside type:

$$-\Delta u + V(x)u = H(u-\beta)f(x,u), \quad x \in \mathbb{R}^N,$$

where $\beta \geq 0$, $N \geq 3$, $V : \mathbb{R}^N \to \mathbb{R}$ is a nonnegative potential which may vanish at infinity, f is a Carathéodory function with subcritical growth, and H denotes the Heaviside function.

The discontinuity in the nonlinearity prevents the direct use of classical variational methods for C^1 functionals. To overcome this difficulty, we employ variational techniques for locally Lipschitz functionals combined with a nonsmooth penalization method inspired by the Del Pino–Felmer approach.

Under suitable assumptions on the potential and the nonlinearity, we prove the existence of at least one positive weak solution of mountain pass type. Moreover, we establish the asymptotic behavior of the family of solutions u_β as $\beta \to 0^+$, proving strong convergence in $D^{1,2}(\mathbb{R}^N)$ toward a positive solution of the limit problem

$$-\Delta u + V(x)u = f(x,u), \quad x \in \mathbb{R}^N.$$

These results extend previous existence results for continuous nonlinearities to the discontinuous framework and contribute to the study of elliptic problems with nonsmooth variational structure.

References

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Strong solutions to singular discontinuous p-Laplacian problems

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The talk is devoted to discuss existence of solutions to a p-Laplacian problem whose reaction is singular (i.e., it blows up when the solution approaches zero) and possesses a null-measure set of discontinuity points. The techniques presented are based on regularization arguments, variational methods, regularity theory, and locality properties. Moreover, a problem exhibiting discontinuous convection terms (i.e., terms depending on the gradient of the solution) will be exposed. In this case, existence of solutions will be obtained via monotonicity techniques.

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Existence results for some elliptic problems in $\mathbb{R}^N$ including variable exponents above the critical growth

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Here, we investigate the existence of solutions for a class of equations involving variable exponents. Our approach requires growth assumptions only near the origin, allowing the nonlinearity to exhibit subcritical, critical, or supercritical Sobolev growth away from this region. To this end, we study two classes of problems: one with subcritical behavior at the origin and another with critical growth at the origin.

We first consider the following class of equations:

$$(\mathcal{P}_1) \qquad -\Delta u + u = |u(x)|^{p(|x|)-1}u(x), \quad x \in \mathbb{R}^N,$$

where p satisfies the following assumption:

(p_1)**:** $p : [0, +\infty) \to (1, +\infty)$ is a continuous function such that $p(0) < 2^* - 1$, where $2^* := \frac{2N}{N-2}$ denotes the critical Sobolev exponent.

Our first main result is stated below.

Theorem 1. *If* (p_1) *holds, then Problem* ($\mathcal{P}_1$) *admits a nontrivial radial weak solution.*

Motivated by the celebrated work of H. Brezis and L. Nirenberg, we next study the existence of weak solutions for the following class of equations:

$$(\mathcal{P}_2) \qquad -\Delta u + u = |u(x)|^{p(|x|)-1}u(x) + \lambda |u(x)|^{q(|x|)-1}u(x), \quad x \in \mathbb{R}^N,$$

where $p, q : [0, +\infty) \to (1, +\infty)$ are continuous functions satisfying the following hypothesis:

(p_2)**:** There exists $r_0 > 0$ such that

$$q(|x|) < p(|x|) = 2^* - 1 \quad \text{for all } x \in B_{r_0}(0).$$

Our second main result reads as follows.

Theorem 2. *Suppose that* (p_2) *holds and that one of the following conditions is satisfied:*

(a) $N \geq 4$ *and* $\lambda > 0$*;*
(b) $N = 3$, $3 < q(|x|) < 5$ *and* $\lambda > 0$*;*
(c) $N = 3$, $1 < q(|x|) \leq 3$ *and* $\lambda > 0$ *is sufficiently large.*

Then Problem ($\mathcal{P}_2$) *admits a nontrivial radial weak solution.*

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On compact embeddings into $\mathrm{L^p}$ and fractional spaces

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Dedicated to Prof. Carlos Tomei on occasion of his 70th birthday.

The study of the fractional Laplacian operator $(-\Delta)^s$ in $\mathbb{R}^N$ with Dirichlet boundary conditions gained enormous momentum through its identification with a Neumann operator in $\mathbb{R}^N \times (0,\infty) = \mathbb{R}^{N+1}_+$, a method mainly introduced by Caffarelli and Silvestre. Since then, several other operators have been studied using this method. In general, a crucial question is attached to this method: is the embedding (in the trace sense) on the ground space $L^q(\mathbb{R}^N)$ compact? This question is very important when dealing with problems of existence of solutions. This paper aims to answer this question for some operators.

Passing to an abstract setting, let X, Y be Hilbert spaces and $\mathcal{A}\colon X \to X'$ a continuous and symmetric operator. We suppose that X is dense in Y and that the embedding $X \subset Y$ is compact. In this paper we show some consequences of this setting for the study of the fractional operator attached to $\mathcal{A}$ in the extension setting $\Omega \times (0,\infty)$ or $\mathbb{R}^{N+1}_+$. Being more specific, we will give some examples where the embedding of the extension domain into $L^2(\Omega)$ is compact, even in the case $\Omega = \mathbb{R}^N$.

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An overview of the regularity theory for the fractional p-Laplacian

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I will describe a number of results obtained in recent years on the regularity theory for solutions of the inhomogeneous fractional p-Laplacian equation. I will restrict in discussing the Hölder continuity properties and focus on the differences rather than on similarities with the classical analogous results pertaining the p-Laplacian. In particular, the boundary regularity in the fractional case involves radically different techniques and statements, mainly due to the nonlocality of the operator. The most recent results in this framework, dealing with unbound suitably summable reaction terms, have been obtained in collaboration with prof. A. Iannizzotto of the University of Cagliari.

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Eigenvalues of the p-Laplacian on general open sets

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We consider the minmax Ljusternik-Schnirelmann levels of the constrained $p-$Dirichlet integral, on a general open set of the Euclidean space. We show that, whenever one of these levels lies below the threshold given by the L^p Poincaré constant “at infinity”, it actually defines an eigenvalue of the Dirichlet $p-$Laplacian. We also prove an exponential decay at infinity for the relevant eigenfunctions: this can be seen as a Šnol-Simon–type estimate for the nonlinear case. Finally, we exhibit some peculiar examples of unbounded open sets to which our main result applies. Based on a joint work in collaboration with L. Brasco and L. Briani.

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Global boundedness and normalized solutions to a p-Laplacian equation

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In the talk I will show the existence of solutions to the p-Laplacian equation

$$-\Delta_p u + (\mathrm{sgn}(p-s) + V(x))|u|^{p-2}u + \lambda|u|^{s-2}u = |u|^{q-2}u \qquad \text{in } \mathbb{R}^N \tag{1}$$

with prescribed $L^s(\mathbb{R}^N)$-norm. Here $N \geq 3$, $p \in [2, N)$, $s \in (1, p)$, $q \in (p\frac{N+s}{N}, \frac{Np}{N-p})$ and $V : \mathbb{R}^N \to \mathbb{R}$ is a suitable potential. We will focus on the radial case and we will see that, even in this case, the proofs can be highly nontrivial. In fact, in some specific cases, they rely on the Pohozaev identity, which is shown to be satisfied by any solution for a large class of potentials (even unbounded) and a global boundedness results for solutions to (1). This is a joint work with R. N. Dhara.

References

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