BOOK OF ABSTRACTS

SS28 - Recent progress on mathematical fluid dynamics

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Organizers:

Milton da Costa Lopes Filho, Federal University of Rio de Janeiro
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SS28

On the self-similar blowup for the generalized SQG equations

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In this talk, we will discuss the locally self-similar blow-up scenario for the inviscid generalized surface quasi-geostrophic equation (gSQG) in $\mathbb{R}^2$. We show that under an L^r growth assumption on the self-similar profile and its gradient, and for apropriate ranges of the self-similar parameter, the profile is either identically zero, and hence blowup cannot occur, or its L^p asymptotic behavior can be characterized for suitable r and p.

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Magnetic relaxation for the MHD equations via the stable manifold method

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We prove that given any sufficiently small and regular solution B of the stationary Euler equations there exists an infinite dimensional family of solutions (u, b) of the non-resistive Magnetohydrodynamics equations (MHD) that relax to $(0, B)$. More precisely, $(u, b) \to (0, B)$ exponentially fast as $t \to +\infty$. This family may be viewed as lying in the stable manifold of the non-resistive MHD equations around the equilibrium state $(0, B)$. As a byproduct of our result, we provide a large class of global regular (small) solutions of the non-resistive MHD equations. Another consequence is that any sufficiently small and regular solution of the stationary Euler equation is topologically *accessible* via MHD according to the definition of Moffatt [1].

References

[1] H. K. Moffatt, *Magnetostatic equilibria and analogous Euler flows of arbitrarily complex topology*, J. Fluid Mech., 159 (1985), 359–378.

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Incompressible 2D Euler with random initial data in the plane

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We consider a random initial vorticity $\omega_0(x) = \sum_{n\in\mathbb{Z}^2} a_n\phi(x-n)$, where ϕ is bounded and compactly supported and $\{a_n\}$ are independent, uniformly bounded, mean 0, variance 1 random variables (in other words, ω_0 is an array of randomly weighted vortex blobs). We prove global well-posedness of weak solutions to the Euler equations in $\mathbb{R}^2$ for almost every such initial vorticity.

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A Liouville theorem for the two-dimensional stationary hypodissipative Navier–Stokes system

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We study the stationary incompressible hypodissipative Navier–Stokes equations on the whole plane $\mathbb{R}^2$, where the dissipation is given by the fractional Laplacian $(-\Delta)^s$. We prove that for $s \in [\frac{1}{3}, 1)$, any solution $u \in \dot{H}^s(\mathbb{R}^2)$ is trivial. This result extends the classical theorem of Gilbarg and Weinberger [1] for the standard dissipative case ($s = 1$). In the range $s \in (0, \frac{1}{3})$, a further integrability assumption is needed.

References

[1] D. Gilbarg, H. F. Weinberger, *Asymptotic properties of steady plane solutions of the Navier-Stokes equations with bounded Dirichlet integral*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4), 5 (1978), 381–404.

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Long-time behaviour of the 1D stochastic Navier-Stokes-Korteweg equations

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We consider a one-dimensional model of compressible viscous fluids in which capillarity effects are not negligible. The dynamics is driven by a stochastic force, and we prove the existence of invariant measures by applying the Krylov-Bogoliubov method. This is achieved in a setting characterized by a non-complete phase space and the lack of standard results concerning continuous dependence on the initial data. The existing literature regarding invariant measures and ergodic properties for compressible fluids is limited to the existence result in [1], where the Navier-Stokes equations are considered with constant viscosity and an adiabatic exponent $\gamma = 1$. Therefore, our result establishes properties for Korteweg fluids that remain unknown for standard compressible fluids in the absence of capillarity. In particular, we prove that the Markov semigroup is Feller, and we account for a wide class of density-dependent viscosities and general capillarity exponents. Furthermore, no restrictions on γ are assumed. The main tools of our analysis involve a BD entropy method combined with a refined stability result based on a relative entropy approach.

References

[1] M. Coti-Zelati, N. Glatt-Holtz, and K. Trivisa, *Invariant measures for the stochastic one-dimensional compressible Navier-Stokes equations*, Appl. Math. Optim., 83, (2021), no. 3, 1487-1522.

[2] D. Donatelli, L. Pescatore and S. Spirito, *Invariant measures for the one-dimensional stochastic Navier-Stokes-Korteweg equations*, Preprint, available at arXiv:2606.07423, (2026).

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Global Behavior of Chemotaxis-Fluid Systems

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This talk addresses a two-dimensional Keller-Segel-Navier-Stokes system describing the interaction between cell density, an attractive chemical consumed by the cells, and a repulsive chemical produced by them. The model incorporates generalized logistic growth, nonlinear consumption and production rates, non-flux boundary conditions for cell and chemical concentrations, and Dirichlet boundary conditions for the fluid velocity. We present results on the existence of global classical solutions under suitable conditions linking chemotactic sensitivities, attraction and consumption rates, and logistic competition effects.

References

[1] D. Barbosa, F. Guillén-González, G. Planas, *Global Classical Solutions for a Two-Dimensional Keller-Segel-Navier-Stokes System of Potential Type*, NoDEA Nonlinear Differ. Equ. Appl. 33 (2026), 61.

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Energy cascade for cross-shear length scales in free-shear three-dimensional incompressible viscous flows

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We present recent results on the energy cascade in free-shear incompressible Newtonian flows driven by an imposed shear profile $\mathbf{U} = (U(z), 0, 0)$ and with mixed boundary conditions: L_x-periodic in the streamwise direction x, L_y-periodic in the spanwise direction y, and free-slip in the shear direction z with height h. We model the fluctuation field $\mathbf{u} = (u, v, w)$ from the shear profile and assume its spatial averages in the cross-shear planes, parallel to xy, vanish. Mean quantities are taken to be ensemble averages with respect to Foias–Prodi stationary statistical solutions. In this setting, we obtain rigorous energy-cascade results as follows.

We decompose the fluctuation field into its Stokes eigenmodes associated with wavenumbers $\kappa = \kappa_{j,l,k} = \sqrt{4\pi^2 j^2/L_x^2 + 4\pi^2 l^2/L_y^2 + k^2\pi^2/h^2}$, $j, l \in \mathbb{Z}$, $j^2 + l^2 \neq 0$, $k \in \mathbb{N}_0$, paying particular attention to the horizontal wavenumbers $\bar{\kappa} = \bar{\kappa}_{j,l} = \sqrt{4\pi^2 j^2/L_x^2 + 4\pi^2 l^2/L_y^2}$, associated with the cross-shear length scales, i.e., perpendicular to the shear direction. We start with the energy-budget relations

$$\nu\kappa_1^3 \langle \|\boldsymbol{\nabla}\mathbf{u}_{\bar{\kappa}_a,\bar{\kappa}_b}\|_{L^2}^2 \rangle + \kappa_1^3 \langle (\!((\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}, \mathbf{u}_{\bar{\kappa}_a,\bar{\kappa}_b})\!)_{L^2} \rangle = -\kappa_1^3 \langle (\!(wU'(z)\mathbf{e}_x, \mathbf{u}_{\bar{\kappa}_a,\bar{\kappa}_b})\!)_{L^2} \rangle,$$

for the flow components $\mathbf{u}_{\bar{\kappa}_a,\bar{\kappa}_b} = \sum_{\bar{\kappa}_a \leq \bar{\kappa} < \bar{\kappa}_b} \mathbf{u}_{\bar{\kappa}}$ with horizontal wavenumbers $\bar{\kappa}_1 \leq \bar{\kappa}_a < \bar{\kappa}_b < \infty$, where $\kappa_1 = \bar{\kappa}_1 = 2\pi/\max\{L_x, L_y\}$ is the smallest possible (horizontal or overall) wavenumber. This balances the energy dissipation term $\epsilon_{\bar{\kappa}_a,\bar{\kappa}_b} = \nu\kappa_1^3 \langle \|\boldsymbol{\nabla}\mathbf{u}_{\bar{\kappa}_a,\bar{\kappa}_b}\|_{L^2}^2 \rangle$ with the shear-production term $\mathcal{P}_{\bar{\kappa}_a,\bar{\kappa}_b} = -\kappa_1^3 \langle (\!(wU'(z)\mathbf{e}_x, \mathbf{u}_{\bar{\kappa}_a,\bar{\kappa}_b})\!)_{L^2} \rangle$ and the energy flux term $\kappa_1^3 \langle (\!((\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}, \mathbf{u}_{\bar{\kappa}_a,\bar{\kappa}_b})\!)_{L^2} \rangle = -\vec{\Pi}_{\bar{\kappa}_a} + \vec{\Pi}_{\bar{\kappa}_b}$, where the fluctuation flux $\vec{\Pi}_{\bar{\kappa}} = -\kappa_1^3 \langle (\!((\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}, \mathbf{u}_{\bar{\kappa},\infty})\!)_{L^2} \rangle$ accounts for the energy transferred per unit time from the whole fluctuation field $\mathbf{u}$ to the horizontal modes larger than or equal to $\bar{\kappa}$. All quantities are per unit mass, with $\rho\kappa_1^{-3}$ as the characteristic mass, and with the density ρ canceling out. For any given wavenumber $\bar{\kappa}$, we define an associated horizontal Taylor-like wavenumber $\mathcal{K}_{\bar{\kappa},\infty} = \left(\langle \|\boldsymbol{\nabla}_{x,y}\mathbf{u}_{\bar{\kappa},\infty}\|_{L^2}^2 \rangle / \langle \|\mathbf{u}_{\bar{\kappa},\infty}\|_{L^2}^2 \rangle \right)^{1/2}$ of the high-wavenumber band above $\bar{\kappa}$, where $\boldsymbol{\nabla}_{x,y} = (\partial_x, \partial_y)$ is the horizontal gradient.

We estimate the shear-production term by exploiting its orthogonality with the horizontal modes and look for an energy cascade in the fluctuation field, across the horizontal scales. We prove two energy-cascade results, one for the energy flux $\vec{\Pi}_{\bar{\kappa}}$ and one for the restricted energy flux $\vec{\Pi}^*_{\bar{\kappa}} = \vec{\Pi}_{\bar{\kappa}} - \vec{\Pi}_\infty$, which discounts possible energy loss to flow singularities, accounted for by the "singularity" flux $\vec{\Pi}_\infty = \lim_{\bar{\kappa}\to\infty} \vec{\Pi}_{\bar{\kappa}}$: We show that $\vec{\Pi}_{\bar{\kappa}} \gtrapprox \epsilon$ and $\vec{\Pi}^*_{\bar{\kappa}} \approx \epsilon$, whenever $\epsilon_{\bar{\kappa}_1,\bar{\kappa}} \ll \epsilon$ and $\kappa_s^2 \ll \mathcal{K}_{\bar{\kappa},\infty}^2$, i.e., for wavenumbers $\bar{\kappa}$ whose low-wavenumber energy dissipation rate is negligible compared with the total energy dissipation rate $\epsilon = \epsilon_{\bar{\kappa}_1,\infty} = \nu\kappa_1^3 \langle \|\boldsymbol{\nabla}\mathbf{u}\|_{L^2}^2 \rangle$ and whose associated horizontal Taylor wavenumber lies above the viscous shear wavenumber $\kappa_s = (S/\nu)^{1/2}$, where $S = \|U'\|_{L^\infty}$ measures the shear gradient. On heuristic grounds, assuming the Kolmogorov energy spectrum holds within the inertial range, these rigorous conditions correspond precisely to an energy cascade in the range from the classical Corrsin scale $\ell_C = (\epsilon/S^3)^{1/2}$ down to the Kolmogorov dissipation scale $\ell_\eta = (\nu^3/\epsilon)^{1/4}$, as expected in the conventional theory of turbulent shear flows.

References

[1] R. Rosa, Energy cascade for cross-shear length scales in free-shear three-dimensional incompressible viscous flows, `https://arxiv.org/abs/2603.14626`, *submitted.*

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Analysis of the θ method for the flow of nonsmooth velocity fields

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In this talk, we report on some recent results regarding numerical approximation of ordinary differential equations with irregular vector fields. Motivated by fluid dynamical settings, we consider Sobolev divergence-free vector fields. We will prove logarithmic rates of convergence (in the mesh size) of the approximating flows constructed via the so-called θ-method towards the unique regular Lagrangian flow of the given vector field. We will also discuss the application to the linear transport equations. The talk is based on a joint work with G. Ciampa (L'Aquila), G. Crippa (Basel), R. D'Ambrosio (L'Aquila), and T. Cortopassi (Pisa).

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