

Some applications of Galois representations in arithmetic geometry

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Galois representations are ubiquitous in arithmetic geometry and have been exploited to prove all manners of (often spectacular) results, including Fermat's last theorem and the finiteness of rational points on curves of genus at least 2. Interesting Galois representations come from considering a commutative algebraic group A , defined over a field K , equipped with a rational point $Q \in A(K)$: the natural Galois action on the set of solutions $P \in A(\bar{K})$ to the equation $nP = Q$ gives useful information about the arithmetic of A ("Kummer theory"). For example, in the special case $Q = 0$, this construction encodes the Galois action on the n -torsion points of $A(\bar{K})$.

After a general overview of the main problems in the area, I will describe some recent results where Galois representations play a major role, either as the object of study or as a crucial ingredient in the proof.