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Nonlinear matrix equations: applications and numerical solution

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A nonlinear matrix equation is an equation of the kind $F(X) = 0$, where $F(X)$ is a matrix-valued function that depends nonlinearly on a square matrix X. Typical examples are polynomial matrix equations, say, $F(X) = A_0 + A_1X + A_2X^2$ or $F(X) = X^p - A$ with p positive integer, power series matrix equations where $F(X) = \sum_{i=0}^{\infty} A_iX^i$, algebraic-type Riccati equations, say, $F(X) = XAX + BX + XC + D$ or $F(X) = X^TAX + BX + X^TC + D$, and more. Such equations underpin many areas of computational science and engineering, including queueing theory [2], control theory [1, 4], fluid mechanics, and structural engineering [3]. The sought solution has a modeling significance and is usually characterized by spectral properties that allow one to determine it, among the many possible solutions.

These equations are in general difficult to solve due to their nonlinear nature and commutativity issues. In this talk, we review numerical methods for their solution based on linearizations. The kind of linearization depends on the nature of the equation. Indeed, polynomial or power series matrix equations are represented in terms of infinite dimensional linear systems, having a block Toeplitz structure. Instead, the solution of algebraic-type Riccati equations is expressed in terms of the deflating subspace of a suitable linear matrix pencil. For the solution of these linearized problems, quadratically convergent algorithms, like cyclic reduction or structured doubling algorithms, are used. Such methods outperform classical Newton's method applied to the equation $F(X) = 0$.

Finally, the numerical solution of quadratic matrix equations with infinite dimensional Toeplitz matrix coefficients is discussed. Such equations arise in the study of random walks in the quarter plane.

Bibliografia

[1] D. Bini, B. Iannazzo, B. Meini, Numerical solution of algebraic Riccati equations. Fundamentals of Algorithms, SIAM, 2011.

[2] D. Bini, G. Latouche, B. Meini, Numerical methods for structured Markov chains, Oxford University Press, 2005.

[3] D. S. Mackey, N. Mackey, C. Mehl, V. Mehrmann, "Structured polynomial eigenvalue problems: Good vibrations from good linearizations", SIAM Journal on Matrix Analysis and Applications, 28 (2006).

[4] V. Mehrmann, The Autonomous Linear Quadratic Control Problem. Lecture Notes in Control and Information Sciences, vol 163. Springer, 1991.

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