

Canonical Metrics in Complex Geometry

Anna Fino
Università di Torino

A natural and important question in complex geometry is finding canonical Riemannian metrics that have special curvature properties. If the complex dimension is one, an answer is provided by the Uniformization Theorem, which says that any Riemann surface can be deformed in a conformal way (i.e. preserving angles) into a Riemann surface of constant curvature. The theorem was proved in 1907 and it can be seen as the one-dimensional answer to the problem in complex geometry, going back to the 1930s, asking to determine which complex manifolds admit Kähler-Einstein metrics. More broadly, in the 1950s Calabi asked whether a compact complex manifold admits a preferred Kähler metric, distinguished by natural conditions on the volume or the Ricci tensor. There has been much interest recently in extending Calabi's Programme to the case of compact complex manifolds which do not admit a Kähler metric, but rather possess "canonical metrics", where the word canonical refers to certain features of the associated fundamental form. In particular, a Hermitian metric on a complex manifold is called pluriclosed if the torsion of the associated Bismut connection is closed, and it is called balanced if its fundamental form is co-closed. In this talk I will survey recent results on pluriclosed and balanced metrics, showing new constructions of compact non-Kähler manifolds and some open problems.