

Transport theory: from functional inequalities to random matrices

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ABSTRACT: The optimal transport problem consists in finding the cheapest way to transport a distribution of mass from one place to another. Apart from its applications to economics, optimal transport theory is an efficient tool to construct change of variables between probability densities, and this fact can be applied for instance to prove stability of minimizers of several geometric/functional inequalities.

More recently, motivated by problems arising in random matrix theory, people have tried to apply these methods in very large dimensions. However the regularity of optimal maps seem to play an important role in this context, and unfortunately one cannot hope in general to obtain regularity estimates that are uniform with respect to the dimension. Based on these considerations, it seems hopeless to apply optimal transport theory in this context. Still, ideas coming from optimal transport can be used to construct approximate transport maps (i.e., maps which send a density onto another up to a small error) which enjoy regularity estimates that are uniform in the dimension, and such maps can then be used to show universality results for the distribution of eigenvalues in random matrices.

The aim of talk is to give an overview of all results.